

A variationally consistent approach for crack propagation based on configurational forces

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Summary: This paper is concerned with a variationally consistent approach suitable for the analysis of cracking in brittle materials. In line with the pioneering works by Griffith, it is assumed that a crack propagates, if this is energetically favorable. However, in order to bypass the well-known defects of Griffith's original idea, a modified non-local criterion is proposed.

Introduction

Since the pioneering work [1] by Griffith has been published, energy-based criteria have been frequently applied to the analysis of crack propagation. The underlying mathematically and physically sound idea is that a crack propagates, if this is energetically favorable. For the derivation of this strategy, Griffith analyzed the force F associated with a (material) variation of a crack tip and postulated that a new crack segment would form, if F reaches a critical threshold, i.e., $F > F_{\text{crit}}$, with F_{crit} denoting a material parameter. Clearly, F represents a material or configurational force, cf. [2]. Unfortunately, Griffith's model shows some problems.

Critical analysis of Griffith method

Griffith's model was critically analyzed in [3; 4]. Francfort & Marigo showed that Griffith's local criterion cannot predict crack initiation, and in its original version, it does neither provide any information concerning the length nor the direction of the new crack segment. Based on this observation, Francfort & Marigo advocated a global criterion. Conceptually identical to the ideas proposed by Griffith, they considered a minimization problem of the type

$$\inf_{\Gamma_c, \varphi} I(\Gamma_c, \varphi), \quad \text{with} \quad I(\Gamma_c, \varphi) = \int_{\Omega} \Psi_{\text{bulk}} \, dV + \int_{\Gamma_c} \Psi_s \, dA. \quad (1)$$

Here, Ψ_{bulk} and Ψ_s are the Helmholtz energy of the bulk and the surface energy associated with the formation of new cracks. Obviously, the total energy depends on the unknown topology of the crack surface Γ_c . According to Eq. (1), crack propagation is understood as a competition between surface energy and energy released in the bulk. Although the global criterion (1) eliminates the aforementioned problems associated with Griffith's theory, it induces some new difficulties. From a mathematical point of view, Eq. (1) represents a so-called free discontinuity problem. The computation of the solution of such a problem is far from being straightforward. Even more importantly, in addition to these technical difficulties, the global energy minimization problem (1) may lead to non-physical minimizers. Such a pathological example was recently given by Larsen, cf. [5]. Larsen analyzed a brittle bar subjected to a one-dimensional (homogeneous) tension stress state. Clearly, if this bar cracks, the energy necessary for forming the crack Γ_c equals $F_c \int_{\Gamma_c} dA$. As a consequence, a crack will develop, if the (constant)

Helmholtz energy of the bulk material reaches the critical threshold

$$\Psi_{\text{bulk}} = \frac{F_c \int_{\Gamma_c} dA}{\int_{\Omega} dV}. \quad (2)$$

Clearly, for a prismatic bar with length l , Eq. (2) results in

$$\Psi_{\text{bulk}} = F_c/l \quad (3)$$

As a result, crack initiation depends on the length of the bar which is obviously not physical.

A non-local crack initiation criterion

Griffith's local criterion and the global counterpart proposed by Francfort & Marigo can be recast into the unified framework

$$\inf_{\Gamma_c, \varphi} \tilde{I}(\Gamma_c, \varphi), \quad \text{with} \quad \tilde{I}(\Gamma_c, \varphi) = \int_{E \subset \Omega} \Psi_{\text{bulk}} dV + \int_{\Gamma_c \subset E} \Psi_s dA. \quad (4)$$

By setting $E = \Omega$ the approach advocated by Francfort & Marigo is obtained, while the limiting case $\text{diam}(E) \rightarrow 0$ corresponds to Griffith's method.

In this contribution, the truly non-local minimization problem (4) with

$$0 < \text{diam}(E) < \text{diam}(\Omega) \quad (5)$$

is analyzed. As a prototype, the model is implemented by employing the so-called Strong Discontinuity Approach (SDA), cf. [6; 7]. In this contribution, it will be shown that the resulting finite element formulation allows to model crack initiation consistently and that the approach avoids the non-physical minimizers predicted by the global model proposed by Francfort & Marigo.

References

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