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THERMO-MECHANICAL MODELLING OF INDIRECT EXTRUSION PROCESS FOR MAGNESIUM ALLOYS

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ABSTRACT: The complexity of deformation phenomena observed in magnesium and its alloys originates from its hexagonal closed packed crystallographic structure. Mechanical anisotropy and asymmetry in tension-compression are hence observed. A phenomenological model derived by Cazacu and Barlat accounts for the respective phenomena. However, the capabilities of this model for simulation of extrusion are limited since strain rate and temperature dependency on flow behaviour are not considered. Modifications including these phenomena have been implemented as user defined subroutine, VUMAT, into the commercial Finite Element (FE) software, ABAQUS/Explicit. Cowper-Symonds' overstress model was chosen to capture the rate dependency of plastic deformation. A fully coupled thermo-mechanical analysis, in which the temperature is assumed as an additional degree of freedom, has been used for calculation of the temperature field by considering heat fluxes and inelastic heat resulting in softening. In order to describe the temperature and rate dependency of deformation, sets of upsetting tests were executed at different punch velocities and test temperatures. Simulations of upsetting tests were performed to fit model parameters by comparing with the corresponding experimental results and then to use as input data for simulations of indirect extrusion trials of cylindrical billets performed at different profile speeds. Temperature measurements during the extrusion trials by a thermocouple embedded in the surface of the die were used to calibrate heat transfer properties between the billet and the die. The FE simulations performed with the proposed model aim at a better understanding of evolution of anisotropy during experimental extrusion trials of magnesium alloys.

KEYWORDS: Rate-dependent- J_3 plasticity, Fully thermo-mechanical coupled analysis, Extrusion, Magnesium alloys

1 INTRODUCTION

Since most metal forming processes in engineering such as extrusion involve complex thermo-mechanical and multiaxial loading conditions resulting in large strains, high strain rates and an increase in temperature due to deformation, a proper yield criterion and hardening law for the description of these processes is an important issue. The most widely applied yield criterion is the one of von Mises [1] for isotropic metals, which is based on the second invariant of the stress deviator. However, it is not suited for hexagonal closed packed metals showing anisotropy in their mechanical behaviour as well as an asymmetric tension/compression behaviour. In order to account for the latter, the centre of the yield surface has to be shifted out of the origin of the deviatoric plane, which requires the third invariant of the stress tensor. A phenomenological model derived by Cazacu and Barlat [2] accounts for the respective phenomena within the frame of rate-independent plasticity. Neither strain rate nor temperature dependency is considered in this model, however.

A modified version of Cazacu and Barlat model was hence developed and implemented as a user defined material (VUMAT) [3], which is applied for fully coupled thermo-mechanical analyses, where temperature and displacement fields are calculated simultaneously.

2 MODELLING

In order to capture strain rate and temperature dependence on plastic yielding, the yield function is written as a function of three internal state variables, namely equivalent plastic strain, $\bar{\epsilon}^{pl}$, plastic strain rate, $\dot{\bar{\epsilon}}^{pl}$, and temperature, θ ,

$$f = \left(J_2^\circ(\bar{\epsilon}^{pl}) \right)^{3/2} - J_3^\circ(\bar{\epsilon}^{pl}) - \tau_y^3(\bar{\epsilon}^{pl}, \dot{\bar{\epsilon}}^{pl}, \theta) \quad (1)$$

where J_2° and J_3° are generalisations of the second and third invariant of the stress tensor for orthotropic materials, containing 6 and 11 model parameters in the three-dimensional case, (i.e. a_k and b_j) respectively. The total number of model parameters reduces to 7 in the

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plane stress case, namely, a_1, a_2, a_3 for J_2° and b_1, b_2, b_3 and b_4 for J_3° .

In order to describe the strain hardening of the material, the parameters a_k and b_j have been assumed to be dependent on the equivalent plastic strain by a saturating exponential law as seen below:

$$\begin{aligned} a_k (k=1,2,3) &= A_k + B_k (1 - \exp(-C_k \bar{\epsilon}^{pl})) \\ b_j (j=1,2,3,4) &= A_j + B_j (1 - \exp(-C_j \bar{\epsilon}^{pl})). \end{aligned} \quad (2)$$

The parameters are identified with the help of a genetic algorithm by minimising the difference between model predictions and the experimentally observed behaviour of the material with respect to the target function,

$$\Psi = \sum_j \sum_k [\mu_{jk}^{\bar{\sigma}} (\bar{\sigma}_{jk}^{set} - \bar{\sigma}_{jk}^{ref})^2 + \mu_{jk}^{\bar{r}} (\bar{r}_{jk}^{set} - \bar{r}_{jk}^{ref})^2]. \quad (3)$$

The target function combines stress, $\bar{\sigma}$, and plastic strain ratio, $\bar{r}$, with weight factors $\mu_{jk}^{\bar{\sigma}}$ and $\mu_{jk}^{\bar{r}}$, respectively.

The yield strength in shear, τ_y , is expressed by the yield strength, σ_y , as measured from the uniaxial compression test by

$$\tau_y = \left(\left(\frac{1}{6} (a_1 + a_3) \right)^{3/2} + \frac{1}{27} (b_1 + b_2) \right)^{1/3} \sigma_y. \quad (4)$$

In order to include the effect of the loading rate into the model, the power law of Cowper and Symonds [4]

$$\frac{\sigma'_y}{\sigma_y} = \left(\frac{\dot{\epsilon}^{pl}}{D(\theta)} \right)^{1/n(\theta)} + 1, \quad (5)$$

was chosen due to being available in the commercial finite element (FE) software, ABAQUS [5]. In this equation, σ'_y is the “dynamic” yield stress at a plastic strain rate $\dot{\epsilon}^{pl}$ and σ_y is the yield stress under quasi-static conditions; D and n are model constants, reference strain rate and exponent, respectively.

To take the variation of the rate dependence with temperature into account, eq. (5) is modified so that D and n are defined as functions of temperature. The model parameters at different temperatures and the corresponding yield stresses are calculated by interpolation of the known values.

2.1 FULLY COUPLED THERMO-MECHANICAL ANALYSIS

In a fully coupled thermo-mechanical analysis [6], temperature is considered as an additionally degree of freedom. The thermal energy balance has to be satisfied. The summation of the rate of heat entering and the rate of energy generated within the body has to be equal to the rate of increase of internal energy, which means

$$\rho \dot{U} = - \frac{\partial}{\partial x} \cdot \underline{q} + r. \quad (6)$$

The negative sign comes from the experimental evidence that heat always flows in the direction of decreasing temperature.

The internal energy rate per unit mass, $\dot{U}$, is expressed by the help of heat capacity, c , as

$$\dot{U} = c \dot{\theta}. \quad (7)$$

For the heat flux per unit volume, r , generated within the body, it was assumed in this study that only plastic deformation has a contribution as

$$r = \eta \underline{\sigma} : \dot{\underline{\epsilon}}^{pl}. \quad (8)$$

η is the inelastic heat fraction to be specified by the user.

Heat conduction is assumed to be governed by the Fourier law as below

$$\underline{q} = -\underline{K} \cdot \frac{\partial \theta}{\partial \underline{x}}, \quad (9)$$

where $\underline{K}$ is the thermal conductivity matrix. Heat convection as one of the boundary conditions applied normal to free surfaces with a surrounding temperature of θ_0 is written as

$$q_n = -h(\theta - \theta_0), \quad (10)$$

where h is called film coefficient.

3 CHARACTERISATION OF METAL FLOW

3.1 TEMPERATURE AND RATE DEPENDENCY

Uniaxial compression or upset tests [7] on specimens are common tests to study metal flow behaviour in extrusion [8,9].

The standard Coulomb friction model has been assumed at the contact. The frictional stress, τ_{fric} , is proportional to the contact pressure, p , as $\tau_{fric} = \mu p$. The friction coefficient, μ , has a value of 0.05 as determined by comparing the barrelling observed in the specimens.

The compression tests on cylindrical specimens with 10 mm of diameter and 15 mm of height machined from as-cast ZEK100 magnesium alloys were performed at three different strain rates, 0.1, 1 and 10 s⁻¹, and three different test temperatures, 300, 400 and 500°C. Temperature was monitored via a thermocouple placed on the surface at the middle of the specimen height and used to fit the parameters required for heat transfer in the FE simulations.

The measured force with respect to the given punch displacement was regenerated with the help of FE simulations. Figure 1 shows the temperature dependent flow behaviour of ZEK100 magnesium alloys obtained by the upsetting tests at a strain rate of 0.1 s⁻¹.

Figure 1: Metal flow at different temperatures at a strain rate 0.1 s^{-1}

Since the stress state is assumed as uniaxial compression, the material model described before was adjusted to mimic conventional von Mises plasticity combined with Cowper-Symonds' power law. The pairs of parameters fitted isothermally to capture the rate dependency are given in Table 1.

Table 1: Model parameters for rate dependency as a variation of temperature

Temperature (°C)	$D \text{ (s}^{-1}\text{)}$	$n \text{ (-)}$
300	800	3
400	400	4
500	50	5

The flow stresses shown in Figure 1 and the model parameters given in Table 1 were used to describe the billet material in simulations of extrusions. The parameter values at different temperatures are calculated by interpolation of the known values.

3.2 ANISOTROPY/ASYMMETRY

As explained before, a genetic algorithm was used to identify the model parameters a_k and b_j by comparing with experimental results. The tensile and compression tests were executed at 300°C and a strain rate of 0.1 s^{-1} on specimens prepared at different orientations with respect to extrusion and transverse directions, ED and TD, respectively. Figure 2 shows the yield loci drawn as isocontours of plastic strain. The respective set of model parameters is labelled as CaBaExpo4 and used for simulations of extrusion.

Since the plastic strain obtained by tensile and compression tests does not cover the whole extrusion process, which results in about 500% plastic strain, the last isostrain curve in Figure 2 has been added by extrapolation. The optimisation procedure was only based on stresses due to lack of information about values of plastic strain ratio, $\bar{r}$.

Figure 2: Optimised yield loci with parameters labelled as CaBaExpo4

4 INDIRECT EXTRUSION

4.1 EXPERIMENTS

Extrusion is a plastic deformation process in which a block of metal (billet) is forced to flow through a die in order to produce profiles with a constant cross section. Extrusion processes can be generally classified with respect to the direction of metal flow, namely direct and indirect extrusion. In this study, indirect extrusion of ZEK100 magnesium alloys is considered so that the die at the front end of the hollow stem moves relative to the container to produce metal flow opposite to the motion of the ram.

The billets were machined down to a diameter of 93 mm and a length of 300 mm for extrusion experiments. Indirect extrusion was carried out at a constant billet temperature of 300°C to produce round bars with a diameter of 17 mm from a container with 95 mm diameter corresponding to an extrusion ratio of 1:30.

The extrusion rate was varied using ram speeds of 5.5 and 11 mm/s. Due to volume constancy, this corresponds to a profile speed of 10 and 20 m/min, respectively. The die was equipped with a thermocouple at the inner surface. This enables the measurement of the profile temperature during the extrusion process.

4.2 SIMULATIONS

Due to symmetry, the mesh for the billet (see Figure 3) was generated with axisymmetric continuum elements having 4 nodes with reduced integration. Since a steady-state was reached in the experimental extrusion trials, a so-called Arbitrary Lagrangian-Eulerian (ALE) formulation [4] was used for the simulations.

In order to keep computational costs reasonable, the container and the die were considered as rigid surfaces, hence no information on deformation or temperature is available on these surfaces. Since there is no relative displacement between the billet and the container, friction between the billet and the container does not occur in indirect extrusion. The contact with the die, on

the other hand, was described by Coulomb friction with a friction coefficient of 0.5.

Figure 3: The mesh used in simulations of extrusion

The heat released from the billet at the contact between billet and the surroundings was calibrated with the help of the thermocouple measurements. The inelastic heat coefficient in eq. (8) was chosen as 0.9, which means that 90% of plastic deformation energy is transformed into heat.

The deformation behaviour of the billet was characterised by the model parameters (CaBaExpo4) as in Figure 2. For comparison, von Mises plasticity was applied as well.

Figure 4: Comparison of simulation results with experiments using ZEK100 at profile speed of 10 m/min

Figure 4 and Figure 5 show that little differences exist in the temperature predictions between the modified Cazacu-Barlat model and the von Mises model. The predictions of forces, however, is significantly improved, for both profile speeds, if the anisotropic yield surfaces of Figure 2 are applied.

Figure 5: Comparison of simulation results with experiments using ZEK100 at profile speed of 20 m/min

5 CONCLUSIONS

A modified version of the Cazacu-Barlat model accounting for strain-rate and temperature dependent hardening has been applied in fully coupled thermo-mechanical numerical analyses has been presented, which provides promising results in the simulations of extrusion for magnesium alloys.

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